Stat 230

Homework Problem Set 5

Due Jun 12 12:30 pm EST.

Please justify your answers and show the steps that lead you to your answer. Without a proper explanation, even a perfectly correct answer will receive a low score. Doubts regarding the problem sets/ Problem setup can be posted on Piazza. Simplify the expressions whenever you can.

Pr. 1 (15 pts)

Mike and Alice meet a cafe at 5 pm. The arrival time of Alice is uniformly distributed 4:45 and 5:15, and the arrival time of Mike is unformly distributed between 4:30 and 5:30.

What is the probability that either of the two has to wait longer than 5 minutes for the other person to arrive?

Pr. 2 (15 pts)

Two discrete random variables X and Y have the following joint distribution:

	$\Pr(X=-1)$	$\Pr(X=0)$	$\Pr(X=1)$
$\Pr(Y=2)$	1/12	1/6	1/12
$\Pr(Y=4)$	1/6	0	1/6
$\Pr(Y=8)$	0	1/3	0

a) Show that X and Y are not independent. (5 pts)

b) Could there be a probability distribution (X', Y') such that all of the following are met (10 pts):

1) The marginal distribution of X' matches that of X;
2) The marginal distribution of Y' matches that of Y;
3) X' and Y' are independent?

Give the distribution of (X', Y') or justify why there cannot be such distribution.

Pr. 3 (20 pts, 5 pts each)

Let P be a random point chosen uniformly at random inside the square with corners $(-1,-1)$, $(-1,1)$, $(1,1)$ $(1,-1)$. Let X be the x-coordinate of P, and Y be the y-coordinate of P.

a) Give a mathematical expression (density) for the joint distribution of (X, Y)

b) What is the probability $X^2 + Y^2 < 1$?

c) What is the probability that $|X + Y| < 2$?

d) What is the probability that $2X - Y < 0$?

Pr. 4 (20 pts)

Two discrete random variables X and Y have the following joint distribution:

(X,Y)		X						
		0	1	2	3	4	5	6
Y	0	c	c	c	c	0	0	0
	1	0	c	c	c	c	0	0
	2	0	0	c	c	c	c	0
	3	0	0	0	c	c	c	c

a) What is the value of c for this to be a well-defined joint probability distribution? (5 pts)

b) Calculate $\mathbb{E}(X), \mathbb{E}(Y), \mathbb{E}(X+Y)$ and $\mathbb{E}(XY)$. (10 pts)

c) Could there be a pair of random variables (X', Y') such that all of the following are met: (5 pts)

1) The expectation $\mathbb{E}(X')$ matches $\mathbb{E}(X)$;
2) The expectation $\mathbb{E}(Y')$ matches $\mathbb{E}(Y)$;
3) The expectation $\mathbb{E}(Y'+X')$ matches $\mathbb{E}(Y+X)$;
4) The expectation $\mathbb{E}(Y'X')$ matches $\mathbb{E}(YX)$;
5) X' and Y' are independent?

Give such distribution or justify why there cannot be such distribution.

Pr. 5 (30 pts, 5 each)

The random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} cxy(1-x), & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

a) What is the value of c for this to be a well-defined probability distribution?

b) Are X and Y independent?

c) Find $\mathbb{E}[X]$.

d) Find $\mathbb{E}[Y]$.

e) Find $\mathrm{Var}(X)$.

f) Find $\mathrm{Var}(Y)$.