Stat 230

Homework Problem Set 2

Due May 22 12:30 pm EST.

Pr. 1 The proportion of spam email in a certain collection of archived email is 0.05. There is a spam filter that correctly classifies spam 95% of the time, but it also mistakes legit email for spam 10% of the time.

    a) What is the probability of the spam filter classifying a randomly chosen email as spam?

    b) Given that the email was classified as spam, what is the probability of being legit?

    c) What is the probability of a legit email ending up classified as spam?

    d) What is the probability of an email being spam and classified as spam?

Pr. 2 You and your friend play a game of rolling 5-sided dice. Whoever rolls higher wins. In case of a draw, both roll again.

You can select among 3 dice (both of you can't select the same). The probabilities of each die landing the numbers 1 to 5 are displayed in the following table:

Dice	Pr(rolls 1)	Pr(rolls 2)	Pr(rolls 3)	Pr(rolls 4)	Pr(rolls 5)
Dice 1	0	$\frac{1}{8}$	$\frac{6}{8}$	$\frac{1}{8}$	0
Dice 2	$\frac{4}{10}$	0	0	$\frac{6}{10}$	0
Dice 3	0	$\frac{6}{10}$	0	0	$\frac{4}{10}$

Your friend *kindly* lets you pick the dice first. Is this a generous offer though?

Pr. 3 A coin is chosen randomly among 2 coins: One is fair and the other is weighted such that the probability of it landing heads is 0.8.

    a) A coin is chosen at random and tossed once. What is the probability of it landing heads?

    b) A coin is chosen at random and the selected coin is tossed 3 times. What is the probability of seeing 2 heads?

    c) You pick a coin (at random) and toss it 5 times. The outcome is {H,T,T,T,H}. Which coin did you pick and how sure are you about that?

Pr. 4 Two people throw a fair coin n times. What is the probability that both of them obtain the same number of heads?

Hint 0.1. *Vandermonde's convolution identity reads:*

$$\binom{m+n}{r} = \sum_{k=0}^{r} \binom{m}{k}\binom{n}{r-k}.$$

The identity is valid for $r, m, n > 0$.

Pr. 5 An airline company estimates 95% of the people that book flights actually show up on the airport to take it.

The company makes (a rather bold) assumption that the individual no-shows are independent, and overbooks a flight whose capacity is 250 with 255 passengers. What is the probability there are enough seats for all the passengers who wish to board the plane?

Pr. 6 The probability that each specific child in a given family with n children will inherit a certain disease is p, independently of each another. Answer the following in terms of n, p and k.

a) What is the probability that exactly k children inherit the disease, given that the first born child has the disease?

b) What is the probability that exactly k children inherit the disease, given that at least one of the children has the disease?