Sta-230: Probability **Summer I 2020**

Key ideas for topic 1: Equally likely outcomes

You should understand the idea of a **sample space and events**. Also start to understand the relation between probability and sets and be able to compute the probability of events.

The naiive definition of probability: If the sample space S (outcome space) is finite, and all the outcomes are equally likely:

$$\Pr(A) = \frac{\#\text{outcomes favorable to } A}{\#\text{outcomes in } S}. \tag{1.1}$$

You should start thinking about **Interpretations of probability**:

> Symmetry: If there are k equally-likely outcomes E, each has $\mathbf{P}(E) = 1/k$;
>
> Frequency: If you can repeat an experiment indefinitely, $\mathbf{P}(E) = \lim_{n\to\infty} \frac{\#E}{n}$;
>
> Belief: If you are indifferent between winning \$1 if E occurs or winning \$1 if you draw a blue chip from a box with 10000 chips of which $100 \times p$ are blue chips, then $\mathbf{P}(E) = p$.

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Key ideas for topic 2: Axioms of Probability

You should know the **axioms of probability**. The probability P of some event E must satisfy a set of rules which were laid out by Kolmogorov (Kolmogorov's axioms):

(1) Nonnegative: $\mathbf{P}(E) \geq 0$

(2) Addition: $\mathbf{P}(E \cup F) = \mathbf{P}(E) + \mathbf{P}(F)$ if $E \cap F = \emptyset$

(3) Countable addition: $\mathbf{P}(\cup_i E_i) = \sum_i \mathbf{P}(E_i)$ if $E_i \cap E_j = \emptyset$ for $i \neq j$. Each E_i is called a countable partition.

(4) Sum to one: $\mathbf{P}(\Omega) = 1$.

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Key ideas for topic 3: Conditional Probability and Independence

Conditional probability: The conditional probability of an event A given an event B is

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(A \cap B)}{\mathbf{P}(B)} = \frac{\mathbf{P}(A, B)}{\mathbf{P}(B)} = \frac{\mathbf{P}(AB)}{\mathbf{P}(B)},$$

or

$$\mathbf{P}(A, B) = \mathbf{P}(B)\,\mathbf{P}(A \mid B).$$

Independence: Events A, B are independent if knowing B gives no information on A and vice versa

$$\mathbf{P}(A \mid B) = \mathbf{P}(A), \quad \mathbf{P}(B \mid A) = \mathbf{P}(B), \quad \mathbf{P}(AB) = \mathbf{P}(A)\mathbf{P}(B).$$

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Key ideas for topic 4: Bayes' Rule, Law of Total Probability

Total probability: Given a partition $\{A_1, ..., A_n\}$ of the state space Ω one can compute the total probability of an event B as

$$\mathbf{P}(B) = \sum_{i=1}^{n} \mathbf{P}(B \mid A_i)\,\mathbf{P}(A_i).$$

Bayes rule: Allows you to compute $\mathbf{P}(A \mid B)$ from $\mathbf{P}(B \mid A)$, $\mathbf{P}(B)$, and $\mathbf{P}(A)$

$$\mathbf{P}(A \mid B) = \frac{\mathbf{P}(B \mid A)\mathbf{P}(A)}{\mathbf{P}(B)}.$$

For the case of a partition $\{A_1, ..., A_n\}$

$$\mathbf{P}(A_i \mid B) = \frac{\mathbf{P}(B \mid A_i)\,\mathbf{P}(A_i)}{\sum_i \mathbf{P}(B \mid A_i)\,\mathbf{P}(A_i)}.$$

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Key ideas for topic 5: Thinking Conditionally

Multiplication rule:

$$\begin{aligned}
\Pr(A_1, A_2, \ldots, A_n) &= \Pr(A_n|A_1, A_2, \ldots, A_{n-1}) \Pr(A_1, A_2, \ldots, A_{n-1}) \\
&= \Pr(A_n|A_1, A_2, \ldots, A_{n-1}) \Pr(A_{n-1}|A_1, A_2, \ldots, A_{n-2}) \Pr(A_1, A_2, \ldots, A_{n-2}) \\
&\vdots \\
&= \Pr(A_n|A_1, A_2, \ldots, A_{n-1}) \ldots \Pr(A_2|A_1) \Pr(A_1)
\end{aligned}$$

Conditional independence: If $\Pr(AB|C) = \Pr(A|C)\Pr(B|C)$, A and B are said to be conditionally independent given C. Conditional independence, in general, does not imply independence nor vice versa.

Pairwise independence: If for A_1, A_2, A_3 $\Pr(A_iA_j) = \Pr(A_i)\Pr(A_j)$ for $i \neq j \in \{1, 2, 3\}$, A_1, A_2, A_3 are said to be pairwise independent. This does not imply A_1, A_2, A_3 are independent, for which we also need $\Pr(A_1, A_2, A_3) = \Pr(A_1)\Pr(A_2)\Pr(A_3)$.