Stat 230

June 19 Demo/ Practice problems

All the problems are ungraded.

Some useful results for Markov chains. For more details, see [1].

For A a subset of the states, the hitting time $T_A(x)$ is the minimal index t such that $(X_t \in A | X_0 = x)$. Note that this is an integer valued random variable (possibly infinite).

The expected hitting time $\tau_A(x)$ is the expected value of $T_A(x)$

Theorem 0.1. *The vector of expected hitting times $\tau_A(x)$ is the smallest non-negative solution to the system of equations:*

$$f(x) = \begin{cases} 1 + \sum_{y \notin A} P(x,y) f(y), & x \notin A \\ 0, & x \in A \end{cases}$$

where $P(x, y)$ is the (single step) transition probability from state x to y.

The hitting probability $h_A(x)$ is the probability that process starting at x ends up in A at some point.

Theorem 0.2. *The vector of hitting probabilities $h_A(x)$ is the smallest non-negative solution to the system of equations:*

$$f(x) = \begin{cases} \sum_{y} P(x,y) f(y), & x \notin A \\ 1, & x \in A \end{cases}$$

where $P(x, y)$ is the (single step) transition probability from state x to y.

Note that Durrett solves these problems with matrices.

Pr.0 Durrett 4.7.40

Pr. 1 Durrett 4.7.39

Pr. 2 Durret 4.7.41

Pr. 3 Durrett 4.7.42

Pr. 4 Durrett 4.7.43

Pr. 5 Durrett 4.7.45

References

[1] David A. Levin, Yuval Peres, Elizabeth L. Wilmer *Markov Chains and Mixing Times 2nd edition* `https://pages.uoregon.edu/dlevin/MARKOV/`