Stat 230

May 13 Demo/ Practice problems

All the problems are ungraded.

Pr. 1 Let X be a continuous random variable with density

$$\begin{cases} \frac{1}{2}, & x \in (0,1) \cup (2,3) \\ 0 & \text{otherwise.} \end{cases}$$

Let Y be the indicator random variable that takes value 1 if $X > 1.5$.

Find the expectation and variance of X using the tower rules.

Pr. 2 A report on Corona virus stated that 80% of the infected cases exhibit only mild to moderate symptoms until recovery, whereas 20% go on to develop severe symptoms and health complications that may result in death or much longer recovery time. Let us hypothesize that the onset time of severe symptoms has an exponential distribution with expectation of 1 week.

What is the probability that an infected individual will not develop severe symptoms within the first week? Given that an infected individual did not develop severe symptoms within the first week, what is the probability that s/he will eventually develop severe symptoms?

To make progress on the above problem, let X denote the unknown disease status of an infected individual ($X = 1$ if severe eventually, or $X = 0$ if mild or moderate until recovery), and let Y (in weeks) denote the time until the onset of sever symptoms. You can take $P(Y = \infty \mid X = 0) = 1$.

Pr. 3 Continuation to problem 2.

Given that an infected individual did not develop severe symptoms within the first week, what is the probability that they will not develop any in the second week?

Pr.4 One of the major difficulties in countering COVID-19 has been the lack of sufficient hospital beds to keep and treat infected patients. Assume that for a patient with the mild/moderate form of infection, the bed occupancy time (terminated by recovery/death) is exponentially distributed with expectation 2 weeks. The same for the severely ill group is also exponentially distributed, but with expectation 5 weeks. Assume the mild vs severe ratio is 80-20.

Assuming that the number of new infected cases tomorrow follows a Poisson distribution with mean 300, what is the expected total hospital bed-weeks this new cohort will occupy? You may assume that the occupancy times of patients are independent of one another.

To make progress, let N denote the number of new cases tomorrow. Then the total hospital bed-weeks occupancy by this new cohort is quantified as $Z = Y_1 + Y_2 + \ldots + Y_N$, where

Y_j is the occupancy time of the j-th new patient. Notice that the number of terms being added here is itself a random variable, namely, N. For example, when $N = 2$, $Z = Y_1 + Y_2$. But when $N = 400$, $Z = Y_1 + Y_2 + \cdots + Y_{400}$. We can take $Z = 0$ if $N = 0$. How would you calculate $\mathbb{E}(Y_j)$ for a given j? How will you calculate $\mathbb{E}(Z \mid N = n)$ for any given $n = 0, 1, 2 \ldots$? What is $\mathbb{E}(Z)$? How about it's variance?