STA230
June 11 Demo/ Practice problems

All the problems are ungraded.

Pr. 1 Pitman 6.2.4
Pr. 2 Pitman 6.2.5
Pr. 3 Pitman 6.2.9(a)(b)
Pr. 4 Pitman 6.2.12
Pr. 5 Pitman 6.3.4

13. Discrete spacings. As in Exercise 12, let $T_1 < \ldots < T_n$ be the places that good elements appear in a random ordering of n good and $N - n$ bad elements. (In terms of a shuffled deck of N cards with n aces, T_i represents the place in the deck where the ith ace lies.) Let $W_1 = T_1 - 1$, the number of bad elements before the first good one. For $2 \le i \le n$, let $W_i = T_i - T_{i-1} - 1$, the number of bad elements between the $(i-1)$th and ith good ones. Let $W_{n+1} = N - T_n$, the number of bad elements after the last good one. Think of the W_i as spacings between the good elements.

a) Find the joint distribution of $W_1, \ldots, W_{n+1}$.

b) Show that the $n+1$ random variables $W_1, \ldots, W_{n+1}$ are exchangeable, hence identically distributed, but not independent.

c) Find a formula for $P(W_i = w)$ for $0 \le w \le N$.

d) Find $E(W_i)$ for $1 \le i \le n+1$ and $E(T_i)$ for $1 \le i \le n$. [*Hint*: Use the symmetry.] Evaluate in the case $N = 52$ and $n = 4$ to find the mean number of cards between any two aces, and the mean position in the deck of the ith ace. (See Chapter 6 Review Exercise 29 for the variance.)

e) Show that for $1 \le i < j \le n+1$ the random variable $W_i + W_j$ has the same distribution as $T_2 - 2$. Deduce from Exercise 12c) a formula for $P(W_i + W_j = t)$ for $0 \le t \le N$.

f) Let $D_n = T_n - T_1 - 1$, the number of elements between the first and last good elements (including the other $n-2$ good ones). Use the result of e) to find a formula for $P(D_n = d)$, $0 \le d \le N$, and find $E(D_n)$.

3.6.13. a) Uniform on all ordered $(n+1)$-tuples of non-negative integers with sum $N - n$ c) $(N-n)_w n/(N)_{w+1}$ d) $E(W_i) = (N-n)/(n+1)$, $E(T_i) = i(N+1)/(n+1)$, 9.6, 10.6, 21.2, 31.8, 42.4 e) $\frac{\binom{n}{1}\binom{N-n}{t}}{\binom{N}{t+1}} \frac{(n-1)}{(N-t-1)}$
f) $P(D_n = d) = P(W_1 + W_{n+1} = N - 2 - d)$. Now use e).
$E(D_n) = \frac{(n-1)(N+1)}{(n+1)} - 1$